B.Sc. Semester-IV Examination, 2022-23

MATHEMATICS [Honours]

Course ID : 42114 **Course Code : SH/MTH/404/GE-4**

Course Title : Differential Equations and Vector Calculus

Time : 2 Hours Full Marks : 40

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meaning.

UNIT-I

1. Answer any **five** from the following questions: 2×5=10

a) Show that the function $f(x, y) = 1 + xy^2$ satisfies a Lipschitz condition w.r. to y on the rectangle $R = \{(x, y): 0 \le x \le 2,\ 0 \le y \le 1\}$. Find a Lipschitz constant.

b) Show that $y = \cos x$ and $y = 2 \sin x + 3 \cos x$ are two linearly independent solutions of the differential equation $\frac{d^2y}{dx^2} + y = 0$.

[Turn Over]

c) Find the particular integral (P.I) of the differential equation $(D^2 - 2D + 1)y = xe^x$ where $D \equiv \frac{d}{dx}$.

d) Find the equilibrium points of $\frac{d^2x}{dt^2} + \frac{dx}{dt} - (x^3 + x^2 - 2x) = 0$.

e) If $\vec{r}(t) = 5t\hat{i} + 2t^2\hat{j} + t^3\hat{k}$, find the value of $\left[\frac{d\vec{r}}{dt}\ \frac{d^2\vec{r}}{dt^2}\ \frac{d^3\vec{r}}{dt^3}\right]$.

f) If $\frac{d^2\vec{r}^2}{dt^2} = \vec{r}$ then show that $\vec{r} \times \frac{d\vec{r}}{dt}$ is a constant vector.

g) If $\vec{\alpha} \times \vec{\beta} + \vec{\beta} \times \vec{\gamma} + \vec{\gamma} \times \vec{\alpha} = 0$ then show that the vectors $\vec{\alpha}, \vec{\beta}, \vec{\gamma}$ are coplanar.

h) Show that $x = 2$ is a regular singular point of the differential equation

$$(x^2 - 4)\frac{d^2y}{dx^2} + 7x\frac{dy}{dx} - 5y = 0.$$

UNIT-II

2. Answer any **four** from the following questions: 5×4=20

a) Solve by the method of variation of parameters:

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-x} \log x.$$

b) Find the general solution of the differential equation $x^2\frac{d^2y}{dx^2}-3x\frac{dy}{dx}+4y=0$ given that $y=x^2$ is a solution of it. 2+3

c) Determine the nature of the equilibrium points of the linear system $\frac{dx}{dt}=10x-y,\ \frac{dy}{dt}=25x+2y$. Also sketch the corresponding phase portrait in the phase plane. 2+3

d) Solve the differential equation $\left(x^3D^3+2x^2D^2+2\right)y=20\left(x+\frac{1}{x}\right)$, where $D\equiv\frac{d}{dx}$.

e) i) A particle moves along the curve $x=2t^2,\ y=t^2-4t,\ z=3t-5$. Find the components of velocity and acceleration at time t = 1, in the direction of $\hat{i}-3\hat{j}+2\hat{k}$.

ii) Prove that, for any vector $\vec{a}$
$\hat{i}\times\left(\vec{a}\times\hat{i}\right)+\hat{j}\times\left(\vec{a}\times j\right)+\hat{k}\times\left(\vec{a}\times\hat{k}\right)=2\vec{a}$.

3+2

f) Find the power series solution of the differential equation $\frac{d^2y}{dx^2}+x\frac{dy}{dx}+x^2y=0$ about x = 0.

UNIT-III

3. Answer any **one** of the following questions: 10×1=10

a) i) Using the method of undetermined coefficients, solve the differential equation

$$\frac{d^2y}{dx^2}+3\frac{dy}{dx}+2y=x+\cos x.$$

ii) Using vector method, test whether the points P(1, 5, -1), Q(0, 4, 5), R(–1, 5, 1) and S(2, 4, 3) are coplanar or not. 6+4

b) i) Solve : $\frac{d^3y}{dx^3}+a^2\frac{dy}{dx}=\sin ax$

ii) Prove that $\vec{a}\times\left(\vec{b}\times\vec{c}\right)=\left(\vec{a}.\vec{c}\right)\vec{b}-\left(\vec{a}.\vec{b}\right)\vec{c}$.

iii) If $\vec{r}\left(t\right)=3t^3\hat{i}+t\hat{j}-2t^2\hat{k}$ then find $\int_0^3\left(\vec{r}\times\frac{d^2\vec{r}}{dt^2}\right)$. 5+3+2

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